U.S. subprime financial crisis contagion on BRIC and European Union stock markets

Daniel Reed Bergmann
Universidade de São Paulo – São Paulo/SP, Brasil

José Roberto Securato
Universidade de São Paulo – São Paulo/SP, Brasil

José Roberto Ferreira Savoia
Universidade de São Paulo – São Paulo/SP, Brasil

Eduardo Augusto do Rosário Contani
Centro Universitário FECAP – São Paulo/SP, Brasil

Contágio da crise norte-americana do subprime sobre os mercados dos BRIC e da União Europeia

A Teoria de Cópula foi utilizada para analisar o contágio entre BRIC (Brasil, Rússia, Índia e China) e mercados de ações da União Europeia com o mercado norte-americano. Os índices de mercado utilizados para o período de 01 de janeiro de 2005 a 27 de fevereiro de 2010 foram: MXBRIC (BRIC), MXEU (União Europeia) e MXUS (Estados Unidos). Avalia-se neste artigo a adequação das principais cópulas encontradas na literatura financeira usando os critérios estatísticos de log-verossimilhança, a informação Akaike e o critério de informação Bayesiana. Apresenta-se um estudo inovador na área de contágio, devido à utilização de cópulas condicionais, que permite calcular o aumento de correlação entre os índices numa abordagem não paramétrica. A cópula condicional simetrizada Joe-Clayton foi a que apresentou o melhor ajuste para os pares de retornos considerados. Os resultados indicam que há evidência do efeito de contágio em ambos os mercados, o da União Europeia e o dos países constituintes do BRIC, para um nível de significância de 5%. Além disso, há evidências de que o contágio da crise financeira nos Estados Unidos foi mais pronunciado na União Europeia do que nos mercados do BRIC para um nível de significância de 5%. Dessa forma, carteiras de ações formadas por empresas dos países do BRIC puderam oferecer maior proteção para os investidores durante a crise financeira do subprime. Esse resultado se alinha a outros estudos que mostram a crescente correlação entre mercados, especialmente em momentos de baixa.

Palavras-chave: contágio, teoria de cópula, correlação, crise do subprime nos Estados Unidos.
1. INTRODUCTION

Globalization, deregulation and technological advances have deeply changed the relationship between the structures of financial markets in different countries. There is sufficient evidence to show that the increasingly faster transmission of information was responsible for a significant portion of greater integration between markets. Faced with these changes, there were questions about the possible disadvantages of this process. Among the many negative aspects pointed out, one of them relates to the intensification of the phenomenon of financial contagion and losses linked to discontinuities in the propagation mechanisms of shocks. The closer relationship between markets can lead to a significant increase in the vulnerability of economies against external financial shocks. Thus, financial contagion raises interest research for economic policy makers and international investors seeking to diversify risks. In the last two decades, the analysis of the patterns of spreading international financial events became the subject of many academic studies focused on volatility models.

Bianconi, Yoshino and Sousa (2013) stated that the financial crisis of 2007-2009 has arguably been the first truly major global crisis since the Great Depression of 1929-1932. While the crisis initially originated in the United States in a relatively small segment of the lending market, i.e. the sub-prime mortgage market, it rapidly spread across virtually all economies, both advanced and emerging, as well as across economic sectors. It also affected equity markets worldwide, with many countries experiencing even sharper equity market crashes than the United States, making it an ideal laboratory to revisit the debate about the presence and sources of contagion in equity markets.

The recent financial crises that occurred in the Latin American economies have raised questions concerning the benefits of diversification, the robustness of domestic financial institutions, and the extent of the domino effect with asymmetries in propagation of contaminations. All these points suggest that the measurement of cross-markets linkages and the assessment of changes in their interdependencies during crises may be crucial for decision-makers such as portfolio managers, central bankers and regulatory authorities.

A central issue in asset allocation and risk management is whether financial markets become more interdependent during financial crises. The importance of this issue grew dramatically during the five major crises of the 1990s. It was the largest recession since the early 1980s. From November 1982 to July 1990 the U.S. economy experienced robust growth, modest unemployment, and low inflation. The Reagan boom rested on shaky foundations, however, and as the 1980s progressed signs of trouble began to mount. On October 19, 1987 stock markets around the world crashed. In the U.S., the Dow Jones Industrial Average lost over 22% of its value. Although the causes of Black Monday were complex, many saw the crash as a sign that investors were worried about the inflation that might result from large U.S. budget deficits. Common to all these events was the fact that the turmoil that originated in one market extended to a wide range of markets and countries in a way that was not possible to explain on the basis of changes in economic fundamentals. The word contagion became popular, both in the press and in the academic literature, to refer to this phenomenon.

According to Horta, Mendes and Vieira (2008), the burst of the U.S. mortgage bubble, in August 2007 is largely recognized as the moment when the international financial markets were stricken by the subprime crisis. Despite the almost generalized interventions by central banks, suggesting that the impact could go global, until then the effects of the crisis were somewhat confined to the U.S.. After the first liquidity injection by the European Central Bank on August 9 that same year, the supply of funds by central banks became a mandatory rule. By providing low cost money, monetary authorities wanted to ensure that commercial banks could maintain a normal level of activity, in spite of the increasing difficulties faced in the interbank money market. In fact, commercial banks were lending each other less frequently and at higher costs, either following an anticipation of losses and the consequent need to maintain adequate levels of reserves, or reflecting the turmoil in the financial system, motivated by the uncertainties on the real dimension of the crisis.

These episodes suggest that the burst of the U.S. mortgage bubble has, in fact, affected all Developed and Latin America markets. In previous crises, contagion effects were visible in stock market indices, and empirical assessments of financial contagion often focus on the dependence among stock market indices in turbulent periods (Bae, Karolyi & Stulz, 2003). Cappiello, Gerard and Manganelli (2005), for instance, suggest that the financial crises that occurred in the 1990s in Asia and Russia affected Latin American markets. Rodriguez (2007) finds evidence of contagion in Asian markets during the 1997 Asian crisis.

An overview of these comprehensive financial crises can be found in Lo (2012). Since financial theories and risk management analysis rely on the dependence structure of assets, the introduction of an alternative measure that overcomes these limitations is paramount. In order to do so, we will employ the Copula Theory, first used by Sklar (1959), which proved that a collection of marginal distributions can be joined by means of a copula to produce their multivariate distribution.

Capturing co-movement between financial asset returns with linear correlation has been the staple approach in modern finance since the birth of Harry Markowitz’s portfolio theory. Linear correlation is the appropriate measure of dependence if asset returns follow a normal multivariate (or elliptical) distribution. However, statistical analysis of the distribution of individual asset returns frequently encounters fat-tails, skewness and other non-normal features. If the normal distribution is not
sufficient then it is not clear how to appropriately measure the dependence between multiple asset returns. Fortunately, the theory of copulas offers a flexible methodology for the general modeling of multivariate dependence. As Cherubini, Luciano and Vecchiato (2004, p. 11) state: “the copula function methodology has become the most significant new technique to handle the co-movement between markets and risk factors in a flexible way”.

According to Cherubini et al. (2004), when one or more marginal distributions for a given asset are non-normal, the traditional measure of correlation (Pearson’s correlation coefficient) is suitable because it is only able to capture linear dependence. As asset returns are typically non-normal and feature non-linear dependence, use of a robust measure of association would be more prudent. Correlation, a broad concept when applied to the fields of finance and insurance, is used as a measure of dependence between random variables. Boyer, Gibson and Mulder (1999) detect pitfalls in the conditional correlation coefficient. They show that conditional correlation, given a selected event or a (large) threshold value, possesses a systematic bias and will differ from the (true) non-conditional correlation coefficient even when the latter is constant. One such measure that is particularly convenient for copula modeling is Kendall’s \( \tau \).

Peng and Ng (2012) state that the reason for using copula models is that they are very flexible and can model correlations as an alternative to normal distribution. It can capture the extreme co-movements (tail dependence) that a simple linear correlation fails to model. Patton (2006) suggested using a dynamic copula approach combined with other evaluation models to measure market dependence. Xu and Li (2009) followed this methodology and use three Archimedean copulas to estimate tail dependence across three Asian futures markets.

Nikoloulopoulos, Joe and Li (2012) employed other copula approaches to study relationships between financial markets but mainly focus on equity indices. Horta et al. (2008) assesses whether the capital markets of developed countries reflect the effects of financial contagion from the U.S. subprime crisis and, if so, whether the intensity of contagion differs across countries. The results in this paper support the evidence for financial contagion, which may reduce the benefits of international portfolio diversification with either equity or volatility products. The dependence structure for both volatility indices and stock indices are asymmetric.

In our study we assessed whether the BRIC (Brazil, Russia, India and China) and European Union stock markets reflect the effects of financial contagion from the U.S. subprime crisis and, if so, whether the intensity of contagion differs across them. We will attempt to answer this research question by applying copula theory. Cherubini et al. (2004) note that there are several types of copula, both conditional and non-conditional, which may be used in dependence modeling, such as the Normal, Student, and Gumbel copulas. The adopted concept of contagion is proposed by Forbes and Rigobon (2002, p. 2223), who posit that financial contagion is “a significant increase in cross-market linkages after a shock to one country (or group of countries)”. Following this, a significant increase in the dependence between the U.S. market (the so-called ground-zero market) and the other markets in the sample analyzed (BRIC and European Union), from the pre-crisis period (i.e. before the subprime mortgage bubble burst) to the crisis period (after the burst), may be interpreted as evidence of contagion. When contagion exists, its intensity across markets is also evaluated.

The goal is to measure dependence between the U.S. index and each of the remaining indices during the pre-crisis and crisis period. Thus, the following pairs of markets are assessed: US-BRIC (US-BRIC) and US-European Union (US-EU).

Rodriguez (2007) explored whether financial crises can be described as periods of change in the dependence structure between markets. He modeled the dependence structure by asymmetries in tail dependence (e.g. lower dependence index). Mendes (2004) says that crises may propagate faster in one direction - a feature which is captured by asymmetric copulas (e.g. symmetrized Joe-Clayton Copula). As with Chan-Lau, Mathieson and Yao (2004), contagion can be defined as the probability of observing large return realizations simultaneously across different financial markets (co-exceedances), rather than as increases in correlations.

There are many papers that deal with financial contagion between stock markets. Serwa and Bohl (2005) apply methods using heteroscedasticity-adjusted correlation coefficients to discriminate between contagion, interdependence and breaks in stock market relationships. Mendes (2004) measured the asymmetry between the markets using copulas.

This article will address the types of copula described in the financial literature and classify copulas according to their goodness of fit. Following Patton (2006), Canela and Pedroira (2012), and Breymann, Dias and Embrechts (2003) goodness of fit was measured by applying the econometric concepts of Log-Likelihood (LL), Akaike’s Information Criterion (AIC), and Bayesian Information Criterion (BIC).

The paper is organized as follows: the second section presents the empirical framework, defining contagion and the different copula types. Section 3 presents and methodology used. The findings and conclusions are presented in Section 4.

2. Contagion and Copula Theory

According to Mendes, Semeraro and Leal (2010), a stationary d-variate process \((X_1, X_2, ..., X_d)_{t \in R} \subset Z\) a set of indices. In our case the joint law of \((X_1, X_2, ..., X_d)\) is independent of \(t\), the dependence structure of \(X = (X_1, X_2, ..., X_d)\) is given by its (constant) copula \(C\). If \(X\) is a continuous random vector with joint cumulative distribution function (c.d.f.) \(F\) with density function \(f\), and marginal c.d.f.s \(F_i\) with
density functions \( f_i, i = 1, 2, \ldots, d \), then a unique copula \( C \) exists, which pertains to \( F \), defined on \([0,1]^d\) such as

\[
C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)) = F(x_1, x_2, \ldots, x_d) \tag{1}
\]

holds or any \((x_1, x_2, \ldots, x_d) \in \mathbb{R}^d\) (SKLAR, 1959). Let \( F_i(x_i) = U_i, i = 1, 2, \ldots, d \). From the assumptions made, \( U_i \) follows a uniform (0,1) distribution. Therefore a copula is a multivariate distribution with standard uniform margins. Multivariate modeling through copulas allows us to factor the joint distribution into its marginal univariate distributions and a dependence structure – its copula. By taking partial derivatives of (1), one obtains:

\[
f(x_1, \ldots, x_d) = c_{1\ldots d}(F_1(x_1), \ldots, F_d(x_d)) \prod_{i=1}^{d} f_i(x_i) \tag{2}
\]

for some d-dimensional copula density \( c_{1\ldots d} \). This decomposition allows us to estimate the marginal distributions \( f_i \) separated from the dependence structure given by the d-variate copula. In practice, this aspect simplifies both specification and estimation of the multivariate distribution.

The copula \( C \) provides all information about the dependence structure of \( F \), regardless of the specification of the marginal distributions. It is invariant under monotone increasing transformations of \( X \), making copula-based dependence measures relevant scale-free tools for studying dependence. For example, to measure monotone dependence (not necessarily linear), one may use Spearman’s rank correlation \( \rho \).

\[
\rho(X_1, X_2) = 12 \int_0^1 u_1 u_2 dC(u_1, u_2) - 3 \tag{3}
\]

The rank correlation \( \rho \) is invariant under strictly increasing transformations. It always exists in the interval \([-1,1]\), does not depend on the marginal distributions; the values +1 and -1 occur when the variables are functionally dependent, that is, when they are modeled by one of the Fréchet limit copulas.

The copula function builds a bridge between the univariate distributions and their multivariate distribution. This justifies the fact that a copula will create dependence alone, as the probability distribution of the random variables involved is given solely by their marginal distributions. The bivariate copulas used in this article are described below. All of the following definitions can be found in Cherubini et al. (2004).

The Gaussian Copula function is:

\[
C_{Go}(v, z) = \Phi_{\rho XY}(\Phi^{-1}(v), \Phi^{-1}(z)) \tag{4}
\]

where \( \Phi_{\rho XY} \) is the joint distribution function of a standard bivariate normal vector with linear correlation coefficient \( \rho XY \); \( \Phi \) is the standard normal distribution function. Therefore,

\[
\frac{1}{2\pi \sqrt{1-\rho^2 XY}} \exp \left( \frac{2\rho XY s t - s^2 - t^2}{2(1-\rho^2 XY)} \right) ds dt \tag{5}
\]

As expression [1] is parameterized by the linear correlation coefficient, it may also be rendered as \( C_{\rho \ Go} \). The following representation, as demonstrated by Roncalli (2001), is equivalent to expression [5]:

\[
C_{\rho \ Go}(v, z) = \int_0^1 \Phi \left( \frac{\Phi^{-1}(z) - \rho_{XY} \Phi^{-1}(v)}{\sqrt{1-\rho_{XY}^2}} \right) dt \tag{6}
\]

The conditional version of [6] may be expressed by:

\[
C_{\rho \ Go}^c(v, z) = \Phi \left( \frac{\Phi^{-1}(z) - \rho_{XY} \Phi^{-1}(v)}{\sqrt{1-\rho_{XY}^2}} \right) \tag{7}
\]

The Gaussian copula generates a multivariate normal distribution if the marginal distributions are standard normal (Sklar, 1959).

Let \( t_\nu : \mathbb{R} \rightarrow \mathbb{R} \) be the univariate Student’s \( t \) distribution with \( I \) degrees of freedom:

\[
t_\nu(x) = \int_0^x \Gamma \left( \left( \frac{\nu + 1}{2} \right) \left( 1 + \frac{s^2}{\nu} \right) \right)^{-\frac{\nu+1}{2}} ds \tag{8}
\]

where \( \Gamma(.) \) is the Euler function. With \( \rho \in I \) and \( \rho_{\nu} \), the bivariate distribution function corresponding to \( t_\nu \) is:

\[
t_{\nu}(x, y) = \int_{-\nu}^x \int_{-\nu}^y \frac{1}{2\pi \sqrt{1-\rho^2}} \left( 1 + \frac{s^2 + t^2 - 2\rho st}{\nu (1-\rho^2)} \right)^{-\frac{\nu+2}{2}} ds dt \tag{9}
\]

The conditional version of [6] may be expressed by:

\[
C_{\nu}(v, z) = \Phi \left( \frac{z - \rho_{\nu} \Phi^{-1}(v)}{\sqrt{1-\rho_{\nu}^2}} \right) \tag{10}
\]

Palaro and Hotta (2006) stated that if the marginal distributions and are two Student-\( t \) distributions with same \( \nu \) degrees of freedom and \( C \) is a Student-\( t \) copula with parameters \( v \) and \( R_{\nu} \), then the bivariate distribution function \( H \), defined by \( H(x, y) = C(F_1(x), F_2(y)) \) is the standardized bivariate \( t \) distribution, with \( \mu = 0 \), linear correlation coefficient \( \rho \) and \( \nu \) degrees of freedom. In this case the \( t \)-copula is the copula function, which joins the marginal t-distributions with equal degrees of freedom to the bivariate t-distribution. The t-Student
Copula generalizes the bivariate t-distribution because it can adopt any marginal distribution.

The Plackett family of copulas is given by:

$$C_\theta(u,v) = \frac{1 + (\theta - 1)(u + v) - \sqrt{(1 + (\theta - 1)(u + v))^2 - 4uv(\theta - 1)}}{2(\theta - 1)}$$ \[11\]

for $\theta = 1$, $C_1(u,v) = uv$, giving the well-known product copula. It is used when two independent random variables are present.

Rockinger and Jondeau (2001) used the Plackett Copula and a dependence measure to ascertain whether the linear dependence varies over time. They worked with returns of European stock market series, the S&P500 index and the Nikkei index. One disadvantage of the Plackett Copula is that it cannot be easily extended for dimensions larger than two.

According to Gumbel (1960):

$$C_\theta(u,v) = \exp\left[-\left\{\frac{1}{\theta} \ln \left(\frac{1}{u} \right)^{\theta} + \ln \left(\frac{1}{v} \right)^{\theta}\right\}\right]$$ \[12\]

The dependence parameter $\theta$ is restricted to the interval $[1, +\infty)$. Values of 1 and $+\infty$ correspond to independence and the Fréchet upper bound, but this copula does not attain the Fréchet lower bound for any value of $0$. Similar to the Clayton Copula, Gumbel (1960) does not allow negative dependence, but in contrast to Clayton, Gumbel exhibits strong right tail dependence and relatively weak left tail dependence. If outcomes are known to be strongly correlated at high values but less correlated at low values, then the Gumbel Copula is an appropriate choice.

According to Frank (1979):

$$C_\theta(u,v) = \frac{1}{\theta} \ln \left(\frac{1 + (\theta u - 1)(\theta v - 1)}{\theta - 1}\right)$$ \[13\]

The dependence parameter $\theta$ may assume any real value ($-\infty$, $+\infty$). Values of $-\infty$, 0, and $+\infty$ correspond to the Fréchet lower bound, independence, and the Fréchet upper bound, respectively. The Frank Copula is popular for several reasons. Firstly, unlike some other copulas, it permits negative dependence between the marginal distributions. Secondly, dependence is symmetric in both tails, similarly to the Gaussian and Student-t Copulas. Thirdly, it is “comprehensive”, in the sense that both Fréchet bounds are included in the range of permissible dependence. Consequently, the Frank Copula can, in theory, be used to model outcomes with strong positive or negative dependence.

According to Clayton (1978):

$$C_\theta(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-\theta}, \quad 0 > \theta > 0$$ \[14\]

As $\theta$ approaches zero, the marginals become independent. As $\theta$ approaches infinity, the copula attains the Fréchet upper bound, but for no value does it attain the Fréchet lower bound. The Clayton Copula cannot account for negative dependence. It has been used to study correlated risks because it exhibits strong left tail dependence and relatively weak right tail dependence. Anecdotal and empirical evidence suggests that loan defaults are highly correlated during periods of recession. Similarly, researchers have studied the “broken heart syndrome” in which spouses’ ages at death tend to be correlated. When correlation between two events, such as performance of two funds or spouses’ ages at death, is strongest in the left tail of the joint distribution, Clayton is an appropriate modeling choice.

Rotation allows the copula to exhibit lower tail dependence, unlike the unrotated Gumbel Copula [12], which only has upper tail dependence. The distribution function is as follows:

$$C(u_n, \ldots, u_1) = \exp\left\{-\sum_{j=1}^n \ln \left(\frac{1}{u_j} \right)^{\theta}\right\}$$ \[15\]

Patton (2006) used a modified Joe-Clayton Copula to model exchange rate returns. The Joe-Clayton Copula is given by:

$$C_{JC}(u,v|\tau_u, \tau_v) = 1 - \left\{\left[\left(1 - (1 - u)^{\gamma}\right)^{\kappa} + \left[1 - (1 - v)^{\kappa}\right]^{\gamma}\right]\right\}^{1/\kappa}$$ \[16\]

with

$$\kappa = 1/\log_2 (2 - \tau_u)$$
$$\gamma = -1/\log_2 (\tau_v)$$
$$\tau_u \in (0,1), \tau_v \in (0,1)$$

The copula has two parameters $\tau_u$ and $\tau_v$, which allow upper tail and lower tail dependence modeling respectively (Patton, 2006). The Joe-Clayton copula still has some slight asymmetry when $\tau_u = \tau_v$, which is not convenient in the financial studies environment. In order to overcome this issue, the copula must be modified, leading to the so-called symmetrized Joe-Clayton Copula:

$$C_{JC}(u,v|\tau_u, \tau_v) = 0, SC_{JC}(u,v|\tau_u, \tau_v) = 0, SC_{JC}(1-u,1-v|\tau_u, \tau_v) = u + v - 1$$ \[17\]

which is symmetrical when $\tau_u = \tau_v$.

Fitting copulas with different tail behavior makes it possible to test whether times of increased dependence can also be characterized by changes in one or both tails of the distribution. However, in order to capture shifts in the dependence structure, the copula that describes it must be time-varying. Patton (2006) pioneered the study of time-varying copulas. He introduced the concept of conditional copula, and applied it to the study of
asymmetries in the dependence structure of a set of exchange rates.

For the bivariate case, Patton (2006) extended the standard definition of the copula to the conditional case. In order to do so the heteroskedasticity pattern, widely used in the financial literature for the volatility of asset returns, was taken into account. Furthermore, many situations require a generalized joint conditional density, such as in the pricing of options with multiple underlying assets, or in the calculation of portfolio VaR.

Without assuming any functional structure, it is obviously impossible to estimate the form of each joint distribution. Therefore, this study assumes that the distribution remains constant over time while some of its parameters vary according to some finite difference equation.

When modeling marginal distributions, Patton (2006) assumes that conditional means evolve according to an autoregressive process, and that the evolution of conditional variances follows a GARCH (1,1) process. The evolution of $C_t$ must also be considered. One may only consider the case in which parameters vary with time (that is, only the functional form of the Conditional Copula remains fixed), or cases in which both the functional form of the copula and its parameters vary. Our Conditional Copula Modeling followed the assumptions outlined by Patton (2006).

Nelsen (1999) shows that any convex linear combination of copulas is also a copula and a time-varying functional form of the conditional copula could, therefore, be a convex sum of several types of copulas.  

2.1. Measures of dependence

A considerable number of concepts underlie the notion of association, some of which will be presented in this section. Among the most commonly used measures of association are: agreement (as distinguished from dependence), linear correlation, tail dependence and positive quadrant dependence. Some measures associated with the above include Kendall’s $\tau$, Spearman’s $\rho$, the linear correlation coefficient and tail dependence parameters.

All measures of dependence are related to the properties of copulas, since by coupling a distribution function to its marginals, a copula “captures certain [...] aspects of the relationship between the variates, from which it follows that [...] positive dependence concepts are properties of the copula” (Nelsen, 1999, p. 29).

2.2. Tail dependence

As the name suggests, measures of tail dependence are used to capture dependence in the tail of a bivariate distribution. They describe the extent to which high (or low) values of a random variable follow the high (or low) values of another random variable. In some cases the concordance between extreme (tail) values of random variables is of interest. For example, one may be interested in the probability that stock indices in two countries exceed (or fall below) given levels. This requires a dependence measure for upper and lower tails of the distribution. Such a dependence measure is essentially related to the conditional probability that one index exceeds a given value given that another exceeds a given value. If such a conditional probability measure is a function of the copula, then it too will be invariant under strictly increasing transformations.

Li and Rose (2009) showed that most investable portfolios have lower tail risk but higher tail dependence than non-investable ones; emerging markets are likely to be more dependent on the world market during large joint losses than large joint gains; and tail dependence of the aggregate and investable markets on the world market varies across countries and regions. Their study employed the skewed Student-t GJR-GARCH model and the SJC Copula.

Let $X$ and $Y$ be continuous random variables with distribution functions $F$ and $G$, respectively. The upper tail dependence parameter $\lambda_U$ is the limit (if it exists) of the conditional probability that $Y$ is greater than the $100t$-th percentile of $G$ given that $X$ is greater than the $100t$-th percentile of $F$ as $t$ approaches 1 from the left, i.e.:

$$\lambda_U = \lim_{t \rightarrow 1^-} P \left[ Y > G^{-1}(t) | X > F^{-1}(t) \right]$$  \[18\]

Likewise, the lower tail dependence parameter $\lambda_L$ is the limit (if it exists) of the conditional probability of $Y$ being less than or equal to the $100t$-th percentile of $G$ given that $X$ is less than or equal to the $100t$-th percentile of $F$ as $t$ approaches 0 from the right, i.e.:

$$\lambda_L = \lim_{t \rightarrow 0^+} P \left[ Y \leq G^{-1}(t) | X \leq F^{-1}(t) \right]$$  \[19\]

These parameters are nonparametric and depend solely on the copula of $X$ and $Y$. According to Rodriguez (2007), intuitively, asymptotic tail dependence is a measure of the propensity of markets to crash (or boom) together, i.e., it can be treated as the contagion effect. As Chan-Lau et al. (2004) posit, Contagion can be defined as the probability of observing large returns simultaneously across different financial markets (co-exceedances) rather than as increases in correlations. So, $\lambda_L$ will be used as a measure of contagion between markets.

Two tests are performed to identify the affected markets and the existence of distinct levels of contagion intensity. The first assesses whether evidence of contagion emerges after the U.S. subprime mortgage bubble burst (the ground-zero market) in August 2007. The second assesses whether the contagion effect is more intense in the BRIC markets than in the European Union market.

**Test 1** – Thus, if contagion does in fact exist, the dependence or co-movement between markets is more intense during a crisis period. Using the lower dependence tail index, $\lambda_L$:  

$$\lambda_L = \lim_{t \rightarrow 0^+} P \left[ Y \leq G^{-1}(t) | X \leq F^{-1}(t) \right]$$  \[19\]


\[
\begin{align*}
H_0: \Delta \lambda_L &= \lambda_L (\text{crisis}) - \lambda_L (\text{pre-crisis}) \leq 0 \\
H_1: \Delta \lambda_L &= \lambda_L (\text{crisis}) - \lambda_L (\text{pre-crisis}) > 0
\end{align*}
\]

There is evidence of contagion if the null hypothesis was rejected for a given level of significance (5%).

**Test 2** – If contagion is more intense in market A than in market B, the increase in dependence between the U.S. market and market A, relevant to the pre-crisis up to the crisis period, is higher than between the U.S. market and market B. Using a lower dependence tail index, the test may be expressed as:

\[
\begin{align*}
H_0: \Delta \lambda_{A-B} &= \{\lambda^A_L (\text{crisis}) - \lambda^A_L (\text{pre-crisis})\} - \{\lambda^B_L (\text{crisis}) - \lambda^B_L (\text{pre-crisis})\} \leq 0 \\
H_1: \Delta \lambda_{A-B} &= \{\lambda^A_L (\text{crisis}) - \lambda^A_L (\text{pre-crisis})\} - \{\lambda^B_L (\text{crisis}) - \lambda^B_L (\text{pre-crisis})\} > 0
\end{align*}
\]

There is evidence of more contagion in market A than B if the null hypothesis was rejected for a given level of significance (5%). The limitation is related to the fact that this test may only be conducted for conditional copulas. Rodriguez (2007) and Horta et al. (2008) did not use conditional copulas.

### 3. DATA AND METHODOLOGY

Horta et al. (2008), state that the pre-crisis period begins on January 1, 2005 and ends immediately before the subprime burst, which is generally regarded as having occurred on August 1, 2007. The crisis period starts at the beginning of August and extends until February 27, 2010, the last day for which data on stock market indices was collected from the Bloomberg platform. Kenourgios, Samitas and Paltalidis (2011) justify the use of weekly data to avoid market microstructure bias at daily frequencies. Thus, weekly closing data on the Morgan Stanley Capital International (MSCI) indices stipulated in U.S. dollars are used for the BRIC and the European Union stock markets.

Table 1 shows the descriptive statistics of the market indices (MXBRC, MXEU and MXUS).

<table>
<thead>
<tr>
<th>Statistics</th>
<th>MXBRC</th>
<th>MXEU</th>
<th>MXUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000915</td>
<td>-0.000169</td>
<td>-0.000357</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.05619</td>
<td>0.032067</td>
<td>0.029801</td>
</tr>
<tr>
<td>Jarque-Bera (Normality)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Observations</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>

**Note:** Mean, standard deviation and Jarque-Bera normality test of the market indices of BRIC, European Union and United States.

- Estimate the parameters of each copula. These parameters refer to both the marginal distributions as the copula that will be used. The LL, AIC and BIC criteria determine the most appropriate statistical copula. The fewer the information criteria for a given copula, the better its statistical fit.
- Estimate lower tail indices for pairs MXBRC/MXUS and MXEU/MXUS in order to verify the impact of contagion on BRIC and European Union stock markets.
- Calculate the confidence intervals for lower tail indices and p-values by the bootstrapping technique. This procedure was also performed by Horta et al. (2008).
- If p-value is lower than 5% for test 1, there is evidence for contagion with a significance level of 5%. Similarly, if p-value is lower than 5% for test 2, contagion is more intense in one market than it is in the other market.

### 4. RESULTS

We assume that the true copula belongs to a given parametric family \( C = \{C_\theta, \theta \in \Theta \} \) with certain mathematical properties, the \( \theta \) estimates obtained by the likelihood method (LM), through optimization of the likelihood function of each copula are then consistent and normally distributed.

Of the several maximum likelihood estimation procedures that can be used in the copula environment, we opted to use the canonical maximum likelihood (CML) method, as did Canela and Pedreira (2012).

According to Roncalli (2001), the CML method estimates association of the copula’s \( \theta \) parameters without taking on any parametric form for the marginal distributions of MXBRC and MXEU returns. The main advantage is that marginal distributions need not be specified, therefore making CML a robust approach that is free of marginal distribution-related specification errors. All algorithms were developed using the MATLAB® software package.
The parameters were estimated for the following copulas: Gaussian, Clayton, Plackett, Frank, Gumbel, rotated Gumbel, Student’s t, Student’s t with time-varying parameters, symmetrized Joe-Clayton (SJC), Gaussian with time-varying parameters, rotated Gumbel with time-varying parameters and symmetrized Joe-Clayton with time-varying parameters (SJC-conditional).

Following the studies of Breymann et al. (2003), Patton (2006), Horta et al. (2008) and Canela and Pedreira (2012), this study used a goodness-of-fit hierarchy through the econometric concepts of log-likelihood (LL), Akaike’s information criterion (AIC) and Bayesian information criterion (BIC). Table 2 and Table 3 shows the results obtained.

Figure 1 presents the evolution of the cumulative returns of the following indices: MXBRIC, MXEU and MXUS between 2000 and 2010.

From 2000 to 2004, cumulative returns of both markets are very similar and after that curves decoupled. We calculated a higher correlation between MXEU and MXUS than between MXBRIC and MXUS during the subprime crisis. The hypothesis tests presented below confirm this intuition.

Our results show that the most appropriate copula for modeling the dependence structure of US/BRIC and US/EU was the symmetrized Joe-Clayton Copula with time-varying parameters. The symmetrized Joe-Clayton copula assumes asymmetric tail dependence, implying that upper and lower tail dependence is not equal, which supports the Markwat, Kole and Van Dijk (2009) Domino Effect Hypothesis. This means that shocks in the contagion channels evolve into global crashes and significantly increase the probability of more severe crashes, similar to a domino effect.

As shown in Tables 2 and 3, dynamic copulas are better suited to asset returns. The most appropriate copula for modeling the dependence structure of US/BRIC and US/EU markets was the symmetrized Joe-Clayton Copula with time-varying parameters. Patton (2006) also found this same copula to be the most appropriate for modeling international exchange rates. The Frank Copula does not fit the data in two tables because the information criteria (LL, AIC and BIC) cannot be calculated. Table 4 shows the statistics for test 1 and test 2.

The above results indicate that there is evidence of contagion effect in both the European Union and BRIC country markets, for a 5% significance level. Thus, U.S. market crashes during the subprime crisis led to significant decreases in the European Union and BRIC markets. Consistent with King

### Table 2

**LL, AIC, and BIC Results for USA/BRIC Country Estimated Copulas**

<table>
<thead>
<tr>
<th>Copulas (USA/BRICs)</th>
<th>LL</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frank</td>
<td>Infinite</td>
<td>Infinite</td>
<td>Infinite</td>
</tr>
<tr>
<td>Student’s t</td>
<td>-210.1533</td>
<td>-432.4127</td>
<td>-430.1325</td>
</tr>
<tr>
<td>SJC</td>
<td>-208.6570</td>
<td>-423.0328</td>
<td>-423.0328</td>
</tr>
<tr>
<td>Gumbel</td>
<td>-207.8312</td>
<td>-421.1892</td>
<td>-421.1892</td>
</tr>
<tr>
<td>Conditional Rotated Gumbel</td>
<td>-206.6843</td>
<td>-420.3905</td>
<td>-420.3905</td>
</tr>
<tr>
<td>Rotated Gumbel</td>
<td>-203.4930</td>
<td>-401.9249</td>
<td>-400.4939</td>
</tr>
<tr>
<td>Plackett</td>
<td>-199.4930</td>
<td>-399.4930</td>
<td>-396.4930</td>
</tr>
<tr>
<td>Conditional Gaussian</td>
<td>-196.0673</td>
<td>-395.4930</td>
<td>-394.930</td>
</tr>
<tr>
<td>Clayton</td>
<td>-192.4930</td>
<td>-390.4930</td>
<td>-390.4930</td>
</tr>
<tr>
<td>Conditional Student’s t</td>
<td>-170.5940</td>
<td>-323.4939</td>
<td>-333.4939</td>
</tr>
<tr>
<td>Gaussian</td>
<td>-165.4934</td>
<td>-320.5493</td>
<td>-330.5939</td>
</tr>
</tbody>
</table>

**Note:** The information criteria (LL, AIC, BIC) show the copula with the highest goodness-of-fit in multivariate distribution formed by the log-returns of the U.S. and the BRIC country log-returns. This procedure was performed by Mendes (2004), Patton (2006) and Canela and Pedreira (2012).
Table 4

Test to Evaluate the Existence of Financial Contagion

<table>
<thead>
<tr>
<th>Test 1 Results</th>
<th>U.S./BRIC</th>
<th>U.S./EU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0805</td>
<td>0.0739</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[0.0513; 0.1035]</td>
<td>[0.0577; 0.0983]</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0042</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

Note: 95% confidence intervals and the p-values calculated using the studentized bootstrapping technique. Since there is no academic literature to determine the sampling distribution of the indices lower tail dependence, which is our measure of contagion, the tests were conducted using the bootstrapping technique.

The results (Table 5) show that extreme negative events (crashes) in the U.S. market tend to have a greater effect on the European Union market than on the BRIC markets with a significance level at 5% (p-value = 0.0001). Thus, there is evidence that the contagion of the U.S. subprime crisis was more pronounced in the European Union than in the BRIC markets.
Table 5
Assessing the Degree of Contagion Between the European Union and BRIC Countries

<table>
<thead>
<tr>
<th>Test 2 Results</th>
<th>$\Delta L^{EU-BRIC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1140</td>
</tr>
<tr>
<td>95% Confidence Interval</td>
<td>[0.1049; 0.1349]</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Note: 95% confidence intervals and the p-values were calculated when applying the studentized bootstrapping technique. Since there is no academic literature to determine the sampling distribution of the index lower tail dependence, which is our measure of contagion, the tests were conducted using the bootstrapping technique. The null hypothesis, the difference between the contagion of the different trade blocks is: $\Delta L^{EU-BRIC} = (\lambda^{EU} - \lambda^{BRIC}) - (\lambda^{EU}_{pre - crisis} - \lambda^{EU}_{crisis}) - (\lambda^{BRIC}_{pre - crisis} - \lambda^{BRIC}_{crisis})$.

5. CONCLUSION

The use of copula allows us to capture how assets relate to each other in moments of greater volatility or crisis. Cherubini et al. (2004) stated that the lack of normality in a random variable is associated with the presence of skewness and/or kurtosis in its marginal distributions. In the multivariate context, the problem of kurtosis may appear through the individual behavior of returns or the influence of large market movements. This concept is known as tail dependence. Intuitively, assets that do not strongly associate with each other on normal trading days may indicate strong association in extreme market moments and vice-versa.

Firstly, our results show that the most appropriate copula for modeling the dependence structure of US/BRIC and US/EU markets was the symmetrized Joe-Clayton Copula with time-varying parameters. The symmetrized Joe-Clayton copula assumes asymmetric tail dependence, implying that upper and lower tail dependence is not equal supporting Domino Effect Hypothesis as Markwat et al. (2009). This means that shocks in the contagion channels evolve into global crashes and significantly increase the probability of more severe crashes, resembling a domino effect.

Secondly, our findings support the evidence of financial contagion, which may reduce the benefits of international portfolio diversification. The results also suggest that correlation (or dependence) increases more (or is at least larger) in bear markets than in bull markets.

Thirdly, we found that crashes in the U.S. market tend to have a greater effect on the European Union market than on BRIC markets (5% significance level). In other words, the contagion of the U.S. subprime crisis was more pronounced in European Union than in BRIC markets.

The dependence structure for both volatility indices and stock indices is asymmetric. Alcock and Hatherley (2009) showed that asymmetric correlation structures (and) do have real economic value in portfolio management. The primary source of this economic value is the ability to better protect portfolio value and reduce the size of any erosion in return relative to the normal portfolio when asymmetric return correlations are accounted for.

Therefore, stock portfolios formed by the BRIC countries would offer greater protection during the subprime crisis. The results of this empirical analysis seem to support the operational advantages associated with definition of contagion proposed by Forbes and Rigobon (2002). In fact, the evidence of increased dependence between countries after the crisis should be carefully considered by portfolio managers as it suggests that a simple strategy of geographical diversification may not always be successful. Furthermore, the results also support the decisions by the central banks to inject liquidity. In theoretical terms, the crisis-contingent theories appear to be the most adequate to explain the transmission of the shock provoked by the U.S. market crisis.

REFERENCES


REFERENCES


U.S. SUBPRIME FINANCIAL CRISIS CONTAGION ON BRIC AND EUROPEAN UNION STOCK MARKETS

The Copula Theory was used to analyze contagion among the BRIC (Brazil, Russia, India and China) and European Union stock markets with the U.S. Equity Market. The market indexes used for the period between January 01, 2005 and February 27, 2010 are: MXBRC (BRIC), MXEU (European Union) and MXUS (United States). This article evaluated the adequacy of the main copulas found in the financial literature using log-likelihood, Akaike information and Bayesian information criteria. This article provides a groundbreaking study in the area of contagion due to
the use of conditional copulas, allowing to calculate the correlation increase between indexes with non-parametric approach. The conditional Symmetrized Joe-Clayton copula was the one that fitted better to the considered pairs of returns. Results indicate evidence of contagion effect in both markets, European Union and BRIC members, with a 5% significance level. Furthermore, there is also evidence that the contagion of U.S. financial crisis was more pronounced in the European Union than in the BRIC markets, with a 5% significance level. Therefore, stock portfolios formed by equities from the BRIC countries were able to offer greater protection during the subprime crisis. The results are aligned with recent papers that present an increase in correlation between stock markets, especially in bear markets.

**Keywords**: contagion, copula theory, correlation, U.S. subprime crisis.

Contagio de la crisis financiera subprime de Estados Unidos sobre los BRIC y la Unión Europea

Se utiliza la Teoría de Cópulas para analizar el contagio entre los BRIC (Brasil, Rusia, India y China) y mercados de acciones de la Unión Europea con el mercado estadounidense. Los índices de mercado utilizados para el período del 1 de enero de 2005 al 27 de febrero de 2010 fueron: MXBRIC (BRIC) MXEU (Unión Europea) y MXUS (Estados Unidos). Se evalúa en este trabajo la adecuación de las principales cópulas encontradas en la literatura financiera mediante el uso de los criterios estadísticos de log-verosimilitud, información de Akaike e información bayesiana. Se presenta un estudio innovador en el área de contagio, debido a la utilización de cópulas condicionales, que permite calcular el aumento de correlación entre los índices, en un enfoque no paramétrico. La cópula Joe-Clayton simetrizada presentó la mejor adecuación para los pares de retornos considerados. Los resultados indican que existe evidencia del efecto de contagio en los mercados de la Unión Europea y de los BRIC, para un nivel de significación del 5%. Además, hay evidencias de que el contagio de la crisis financiera de Estados Unidos fue más pronunciado en la UE que en los mercados de los BRIC para un nivel de significación del 5%. De esa manera, las carteras de acciones formadas por empresas de los países BRIC pudieron ofrecer una mayor protección a los inversores durante la crisis financiera subprime. Este resultado está en línea con otros estudios que muestran la creciente correlación entre los mercados, especialmente en los momentos de caída.

**Palabras clave**: contagio, teoría de cópulas, correlación, crisis subprime de Estados Unidos.